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April 9, 1979

Dear Neil:

Thanks for the addenda to The Handbook.

Because of my young friend Bob Donaghey, I have been looking at what he calls the Motzkin numbers: Handbook No. 456. Besides a gaggle of identities, I have found the recurrence ($m_0 = 1$)

$$(n+2)m_n = (2n+1)m_{n-1} + 3(n-1)m_{n-2} \quad n = 1, 2, \dots$$

The derivation uses

$$1 = (1-2x)c(x) + (1-4x)xc'(x) \quad - \text{prime} = \text{derivative}$$

and

$$c(x) = 1 + ym(y), \quad y = \frac{x}{1-x} \quad (\text{one of the identities})$$

Of course $c(x) = \sum_0 c_n x^n$, $c_n = \text{Catalan no} = \frac{1}{n+1} \binom{2n}{n}$, and $m(x) = \sum m_n x^n$, $m_n = \text{Motzkin no.}$

I have not been able to look at your reference (for 456) JSIAM 17, 254-69 and suppose it is possible that the recurrence is already known there. It would help me if your files contain the author and title of this paper, but I can live without this information.

Give my regards to Ann

John

P.S. I feel very fond of Ma Bell now that my pension is raised 8.1%!!

John Riordan